

Modern Physics Letters A  
© World Scientific Publishing Company

## Correspondence Between DGP Brane Cosmology and $5D$ Ricci-flat Cosmology

YONGLI PING\*, LIXIN XU and HONGYA LIU†

*School of Physics and Optoelectronic Technology,  
Dalian University of Technology, Dalian, Liaoning 116024, P.R.China*

*\*ylping@student.dlut.edu.cn*

*†hyliu@dlut.edu.cn*

Received  
Revised

We discuss the correspondence between the DGP brane cosmology and  $5D$  Ricci-flat cosmology by letting their metrics equal each other. By this correspondence, a specific geometrical property of the arbitrary integral constant  $I$  in DGP metric is given and it is related to the curvature of  $5D$  bulk. At the same time, the relation of arbitrary functions  $\mu$  and  $\nu$  in a class of Ricci-flat solutions is obtained from DGP brane metric.

*Keywords:* DGP brane; STM theory; cosmology.

PACS numbers: 04.50.+h, 98.80.-k, 02.40.-k

### 1. Introduction

An increasing number of people believe that our universe is a  $5D$  spacetime which is a  $4D$  spacetime with an extra dimension, such as brane theory<sup>1,2,3,4,5,6,7,8</sup> and Space-Time-Matter (STM) theory.<sup>9,10</sup> In the brane world model, the gravity can freely propagate in all dimensions, while the standard matter particles and forces are confined on the 3-brane. The  $5D$  brane world with modified gravity is proposed by Dvali, Gabadadze and Porrati (DGP).<sup>11,12,13</sup> There are the brane and bulk Einstein terms in the action of DGP model. It was shown that the DGP model allows for an embedding of the standard Friedmann cosmology in the sense that the cosmological evolution of the background metric on the brane can entirely be described by the standard Friedmann equation plus energy conservation on the brane.<sup>14,15</sup> And Dick gives an exact metric of  $5D$  bulk. Moreover DGP brane model is used to explain accelerating expansion of universe.<sup>16,17,18,19,20,21</sup> A comprehensive review on DGP cosmology is dished up in Ref. 22.

In STM theory, the  $5D$  manifold is Ricci-flat with  $R_{AB} = 0$  while the  $4D$  matter is induced from the  $5D$  vacuum. This theory is supported by the Campbell-Magaard theorem<sup>23,24</sup> which states that any analytic solution of Einstein's equations in  $N$ -dimensions can be locally embedded in a  $(N + 1)$ -dimensional Ricci-flat manifold.

2 *Y. Ping et al.*

A class of 5D Ricci-flat cosmological solutions was firstly presented by Liu and Mashhoon<sup>25</sup> and restudied latter by Liu and Wesson.<sup>26</sup> This class of solutions is algebraically rich because it contains two arbitrary functions of the time  $t$ . the solutions are utilized in cosmology<sup>27,28,29,30,31,32,33,34,35</sup> and are relate to the brane model.<sup>36,37,38</sup>

In this paper, we discuss the correspondence between DGP brane cosmology and 5D Ricci-flat cosmology. By studying the solutions in DGP model and 5D Ricci-flat cosmology, a clear geometrical property of the Integral constant  $I$  is obtained in DGP brane, and also a constraint is given on a class of solutions in STM theory. Then the evolution of the scale factor  $a(y, t)$  and the condition where  $a(y, t)$  has a bounce are discussed.

## 2. DGP Brane cosmology and 5D Ricci-Flat cosmology

We consider a general five-dimensional spacetime metric. Because we are interested in cosmological solutions, the metric is taken as

$$dS^2 = -n^2(y, t)dt^2 + a^2(y, t)\gamma_{ij}dx^i dx^j + b^2(y, t)dy^2, \quad (1)$$

where,  $\gamma_{ij}$  is a maximally symmetric 3D metric where  $k = 0, \pm 1$  parameterizes the spatial curvature. Adopting the Gaussian normal system gauge  $b(y, t) = 1$ , the 5D Einstein tensors  $\tilde{G}_{AB}$  are

$$\tilde{G}_{00} = 3n^2 \left( \frac{\dot{a}^2}{n^2 a^2} - \frac{a'^2}{a^2} + \frac{k}{a^2} \right) - 3n^2 \frac{a''}{a}, \quad (2)$$

$$\tilde{G}_{ij} = \left( \frac{a'^2}{a^2} - \frac{\dot{a}^2}{n^2 a^2} - \frac{k}{a^2} \right) \gamma_{ij} + 2 \left( \frac{a''}{a} + \frac{n'a'}{na} - \frac{\ddot{a}}{n^2 a} + \frac{\dot{n}\dot{a}}{n^3 a} \right) \gamma_{ij} + \frac{n''}{n} \gamma_{ij}, \quad (3)$$

$$\tilde{G}_{05} = 3 \left( \frac{n'\dot{a}}{na} - \frac{\dot{a}'}{a} \right), \quad (4)$$

$$\tilde{G}_{55} = 3 \left( \frac{a'^2}{a^2} - \frac{\dot{a}^2}{n^2 a^2} - \frac{k}{a^2} \right) + 3 \left( \frac{n'a'}{na} + \frac{\dot{n}\dot{a}}{n^3 a} - \frac{\ddot{a}}{n^2 a} \right). \quad (5)$$

Firstly, in DGP brane cosmology, the five-dimensional Einstein equations take the form as

$$\tilde{G}_{AB} = \tilde{R}_{AB} - \frac{1}{2}\tilde{R}\tilde{g}_{AB} = \kappa_{(5)}^2 \tilde{S}_{AB}, \quad (6)$$

where  $\tilde{R}_{AB}$  is the 5D Ricci tensor,  $\tilde{R} = \tilde{g}^{AB}\tilde{R}_{AB}$  is the scalar curvature,  $\kappa_{(5)}$  is related to the 5D Newton's constant  $G_{(5)}$  and  $\kappa_{(5)}^2 = 8\pi G_{(5)} = M_{(5)}^{-3}$  where  $M_{(5)}$  is the 5D Planck mass. The stress-energy-momentum tensor  $\tilde{S}$  contains three parts,

$$\tilde{S}_B^A = \tilde{T}_B^A|_{bulk} + T_B^A|_{brane} + \tilde{U}_{AB}, \quad (7)$$

where  $\tilde{T}_B^A|_{bulk}$  and  $T_B^A|_{brane}$  are the energy momentum tensor of bulk and brane respectively and  $\tilde{U}_{AB}$  is contribution coming from the scalar curvature of the brane.

$\check{T}_B^A|_{bulk}$  and  $T_B^A|_{brane}$  are written as

$$\check{T}_B^A|_{bulk} = diag(-\rho_B, P_B, P_B, P_B, P_B), \quad (8)$$

$$T_B^A|_{brane} = \delta(y)diag(-\rho_b, p_b, p_b, p_b, 0), \quad (9)$$

where the energy density  $\rho_B$  and pressure  $P_B$  of the bulk are independent of the coordinate  $y$ ; the energy density  $\rho_b$  and pressure  $p_b$  are the function of time.

For  $\check{T}_{05} = 0$ , Eq. (4) implies

$$\frac{n'}{n} = \frac{\dot{a}'}{\dot{a}}. \quad (10)$$

From Eqs. (2), (5) and (6), they are obtained<sup>39</sup>

$$\frac{\partial}{\partial y} \left( \frac{\dot{a}^2}{n^2} a^2 - a'^2 a^2 + k a^2 \right) = \frac{2}{3n^2} a' a^3 \kappa^2 \rho_B, \quad (11)$$

$$\frac{\partial}{\partial t} \left( \frac{\dot{a}^2}{n^2} a^2 - a'^2 a^2 + k a^2 \right) = \frac{2}{3} \dot{a} a^3 \kappa^2 P_B. \quad (12)$$

Assuming there is nothing in the bulk, they have  $\rho_B = 0$  and  $P_B = 0$ . Then, Eqs. (11) and (12) are rewritten as<sup>14,15</sup>

$$\frac{\partial}{\partial y} \left( \frac{\dot{a}^2}{n^2} a^2 - a'^2 a^2 + k a^2 \right) = 0, \quad (13)$$

$$\frac{\partial}{\partial t} \left( \frac{\dot{a}^2}{n^2} a^2 - a'^2 a^2 + k a^2 \right) = 0, \quad (14)$$

and

$$I^+ = \left( \frac{\dot{a}^2}{n^2} - a'^2 + k \right) a^2 \Big|_{y>0}, \quad (15)$$

$$I^- = \left( \frac{\dot{a}^2}{n^2} - a'^2 + k \right) a^2 \Big|_{y<0}, \quad (16)$$

are two constants. In Ref. [14, 15], the author find that the standard Friedmann equation holds on the brane is equivalent to the smoothness condition,

$$\lim_{\epsilon \rightarrow +0} a' \Big|_{y=\epsilon} = \lim_{\epsilon \rightarrow +0} a' \Big|_{y=-\epsilon}, \quad (17)$$

which means that  $a$  is smooth across the brane. So, this leads to  $I^+ = I^- = I$ .

In order to simplify the previous equations, the gauge  $n(0, t) = 1$  is adopted. Then, from Eq. (10),  $n(y, t)$  is obtained as

$$n(y, t) = \frac{\dot{a}(y, t)}{\dot{a}(0, t)}. \quad (18)$$

By substituting  $n(y, t)$  into Eq. (15), the constant  $I$  is rewritten as

$$I = (\dot{a}^2(0, t) - a'^2(y, t) + k) a^2(y, t). \quad (19)$$

4 *Y. Ping et al.*

Choosing  $a'(y, t) > 0$  for the sign of  $y$  in the direction of increasing scale factor, from Eqs. (19) and (18), they are obtained as

$$a^2(y, t) = a^2(0, t) + (\dot{a}^2(0, t) + k)y^2 + 2\sqrt{(\dot{a}^2(0, t) + k)a^2(0, t) - I}y, \quad (20)$$

$$n(y, t) = \left[ a(0, t) + \ddot{a}(0, t)y^2 + a(0, t)y \frac{a(0, t)\ddot{a}(0, t) + \dot{a}^2(0, t) + k}{\sqrt{(\dot{a}^2(0, t) + k)a^2(0, t) - I}} \right] \times \left[ a^2(0, t) + (\dot{a}^2(0, t) + k)y^2 + 2\sqrt{(\dot{a}^2(0, t) + k)a^2(0, t) - I}y \right]^{-1/2} \quad (21)$$

The metric components lead to an exact solution of 5D spacetime. We can find the solution is similar to the bounce solution which is derived from STM theory.<sup>25,26</sup>

In the STM theory for the 5D Ricci-flat cosmology, the Ricci tensor are  $\tilde{R}_{AB} = 0$ . Therefore, in term of the 5D Einstein tensor  $\tilde{G}_{AB} = \tilde{R}_{AB} - \frac{1}{2}\tilde{R}\tilde{g}_{AB}$ , they are

$$\tilde{G}_{AB} = 0. \quad (22)$$

The 4D Einstein field equations are given as

$$G_{\mu\nu} = T_{\mu\nu}. \quad (23)$$

The central idea of STM theory is that (23) is a subset of (22) with the induced 4D energy-momentum tensor  $T_{\mu\nu}$  which has the classical properties of matter.

A class of 5D Ricci-flat cosmological solutions in STM theory reads<sup>26</sup>

$$dS^2 = B^2 dt^2 - A^2 \left( \frac{dr^2}{1 - kr^2} + r^2 d\Omega^2 \right) - dy^2, \quad (24)$$

$$A^2 = (\mu^2 + k)y^2 + 2\nu y + \frac{\nu^2 + K}{\mu^2 + k}, \quad (25)$$

$$B = \frac{1}{\mu} \frac{\partial A}{\partial t} \equiv \frac{\dot{A}}{\mu}, \quad (26)$$

where  $d\Omega^2 = d\theta^2 + \sin^2 \theta d\psi^2$ ,  $\mu = \mu(t)$  and  $\nu = \nu(t)$  are two arbitrary functions of time  $t$ ,  $k$  is the 3D curvature index ( $k = \pm 1, 0$ ), and  $K$  is a constant. Because the 5D manifold (24)-(26) is Ricci-flat, we have  $I_1 \equiv R = 0$ ,  $I_2 \equiv R^{AB}R_{AB} = 0$ , and

$$I_3 \equiv R^{ABCD}R_{ABCD} = \frac{72K^2}{A^8}, \quad (27)$$

so  $K$  is related to the 5D curvature.

Comparing (20) with (25), we find if

$$\mu = \dot{a}(0, t), \quad (28)$$

$$\nu = \sqrt{(\dot{a}^2(0, t) + k)a^2(0, t) - I}, \quad (29)$$

$$a^2(0, t) = \frac{\nu^2 + K}{\mu^2 + k}, \quad (30)$$

the bulk solutions will be the same as the Ricci-flat solutions. Meanwhile, we can obtain  $I = K$ . In brane theory, the  $I$  in (20) is only an arbitrary integral constant, while  $K$  is related to the 5D curvature in the Ricci-Flat cosmology. Therefore, the

constant  $I$  has a new geometrical property in the brane theory. The constant  $I$  is related to the 5D curvature of the 5D bulk. In the Ricci-flat solutions, by Eqs. (28) and (30), the relation of arbitrary functions  $\mu$  and  $\nu$  is given

$$\mu(\nu^2 + K)^{1/2}(\mu^2 + k)^{3/2} = \nu\dot{\nu}(\mu^2 + k) - \mu\dot{\mu}(\nu^2 + K). \quad (31)$$

For  $K = 0$ , the 5D Kretschmann invariant in (27) will be  $I_1 = I_2 = I_3 = 0$ . In the braneworld,  $K = 0$  leads to  $I = 0$ . In this case the components of metric in Eqs. (20) and (21) are

$$a(y, t) = a(0, t) + \sqrt{\dot{a}^2(0, t) + k}y, \quad (32)$$

$$n(y, t) = 1 + \frac{\ddot{a}(0, t)}{\sqrt{\dot{a}^2(0, t) + k}}y. \quad (33)$$

When  $k = 0$ , the Eqs. (32) and (33) become

$$a(y, t) = a(0, t) + \dot{a}(0, t)y, \quad (34)$$

$$n(y, t) = 1 + \frac{\ddot{a}(0, t)}{\dot{a}(0, t)}y. \quad (35)$$

With the relations  $\mu = \dot{a}(0, t)$  and  $a(0, t) = \nu/\mu$  where  $k = K = 0$ , Eqs. (34) and (35) are expressed with  $\mu$  and  $\nu$  as

$$a(y, t) = \frac{\nu}{\mu} + \mu y, \quad (36)$$

$$n(y, t) = 1 + \frac{\dot{\mu}}{\mu}y. \quad (37)$$

When  $k = 0$  and  $K = 0$ , the relation Eq. (31) becomes

$$\mu^3 = \dot{\nu}\mu - \dot{\mu}\nu, \quad (38)$$

and the components of the 5D Ricci-flat metric (25) and (26) are rewritten as

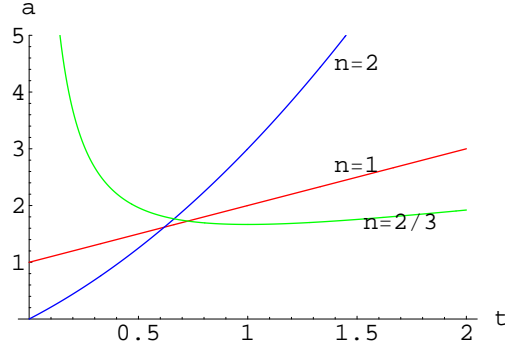
$$A(y, t) = \frac{\nu}{\mu} + \mu y, \quad (39)$$

$$B(y, t) = 1 + \frac{\dot{\mu}}{\mu}y. \quad (40)$$

Obviously, they have the same forms as the ones in brane. However they have different motivation. The main difference is that authors adopt different points of view regarding the matter content and dynamics of 4D spacetime in brane and STM theory.

### 3. The evolution of scale factor $a(y, t)$

In the DGP model, only one brane is contained and put at the position of  $y = 0$ , where the evolution of brane universe is described by the scale factor  $a(0, t)$ . And the bulk solution is derived as the equation (20). However, in the STM theory, our universe is a hypersurface  $y = \text{constant}$ . So, there is a correspondence between the solution of DGP model and STM theory. In details, once the evolution  $a(0, t)$  on the

Fig. 1. Evolution of the scale factor  $a(y, t)$  with  $y = 1$ ,  $n = 2/3, 1, 2$ .

DGP brane is set, the evolution of STM universe is described by the scale factor  $a(y, t)$ . For example, we let  $a(0, t) = t^n$  on the brane, and then derive the scale factor in the STM theory as

$$a(y, t) = t^n + nt^{n-1}y, \quad (41)$$

$$n(y, t) = 1 + (n-1)t^{-1}y. \quad (42)$$

We calculate the first and second derivative of  $a(y, t)$  with respect to time  $t$  and get

$$\dot{a}(y, t) = nt^{n-1} + n(n-1)t^{n-2}y, \quad (43)$$

$$\ddot{a}(y, t) = n(n-1)t^{n-2} + n(n-1)(n-2)t^{n-3}y. \quad (44)$$

Considering the Eqs. (43) and (44), if there is a bounce on  $y = \text{constant}$  supersurface in STM cosmology, the necessary condition that  $a(y, t)$  has the extremum is  $\dot{a} = 0$  and  $\ddot{a}(0, t) > 0$ . In contrast, a non-bounce cosmology is  $\dot{a} \neq 0$  and  $\ddot{a} \geq 0$ . When  $\dot{a} = 0$ , from Eq. (43), we have

$$t_b = -(n-1)y. \quad (45)$$

For  $\ddot{a} > 0$ , from Eq. (44), we let  $y > 0$  and get  $0 < n < 1$ . Therefore,  $a(y, t)$  should have minimum when  $0 < n < 1$  and we name it as a bounce. On the contrary when  $t \neq -(n-1)y$ , there is non-bounce. We plot the evolution of the scale factor with  $y = 1$  when  $n = 1$ ,  $n > 1$  and  $n < 1$  respectively. Fig.1 shows the evolution of the scale factor  $a(y, t)$  with different values of  $n$  and  $y = 1$ .

#### 4. Conclusions

In this paper, we have studied the correspondence between DGP brane cosmology and 5D Ricci-flat cosmology by researching their exact solutions. The solutions given by Dick in DGP brane are very similar to the bounce solutions in Ricci-flat cosmology. Contrasting these two solutions we find these two solutions have the same form when the constant  $I$  in DGP brane and  $K$  in Ricci-flat cosmology satisfy

$I = K$  and the arbitrary functions  $\mu$  and  $\nu$  satisfy (31). Therefore, in this way, the arbitrary integral constant  $I$  is endowed with specific geometrical property and it is related to the curvature of 5D bulk. At the same time, the relation of the arbitrary function  $\mu$  and  $\nu$  in Ricci-flat cosmology is obtained. Finally, the evolution of the scale factor  $a(y, t)$  in STM cosmology is discussed by giving  $a(0, t)$  on the DGP brane. Let  $a(0, t) = t^n$  in the brane, the evolution of  $a(y, t)$  in STM cosmology is determined by  $n$  with  $y = \text{constant}$ . We obtain the necessary condition which  $a(y, t)$  has the bounce is  $0 < n < 1$  and  $y > 0$ . In Fig.1, the evolution of the scale factor  $a(y, t)$  with  $y = 1$  is shown with different values of  $n$ , where  $n = 1$  corresponds to an uniform speed expansion universe,  $n \geq 1$  checks with no bounce, while  $n < 1$  squares with a bounce universe.

### Acknowledgments

This work was supported by NSF (10573003), NSF (10647110), NBRP (2003CB716300) of P. R. China and DUT 893321.

### References

1. N. Arkani-Hamed, S. Dimopoulos, G. Dvali, *Phys. Lett. B* **429** 263 (1998), hep-ph/9803315.
2. N. Arkani-Hamed, S. Dimopoulos, G. Dvali, *Phys. Rev. D* **59** 086004 (1999), hep-ph/9807344.
3. I. Antoniadis, N. Arkani-Hamed, S. Dimopoulos, G. Dvali, *Phys. Lett. B* **436** 257 (1998), hep-ph/9804398.
4. P. Horava, E. Witten, *Nucl. Phys. B* **460** 506 (1996), hep-th/9510209.
5. E. Witten, *Nucl. Phys. B* **471** 135 (1996), hep-th/9602070.
6. P. Horava, E. Witten, *Nucl. Phys. B* **475** 94 (1996), hep-th/9603142.
7. L. Randall, R. Sundrum, *Phys. Rev. Lett.* **83** 3370 (1999), hep-ph/9905221.
8. L. Randall, R. Sundrum, *Phys. Rev. Lett.* **83** 4690 (1999), hep-th/9906064.
9. P.S. Wesson, *Space-Time-Matter* (World Scientific, Singapore, 1999).
10. J.M. Overduin and P.S. Wesson, *Phys. Rep.* **283**, 303 (1997), gr-qc/9805018.
11. G. Dvali, G. Gabadadze, M. Porrati, *Phys. Lett. B* **485** 208 (2000), hep-th/0005016.
12. G. Dvali, G. Gabadadze, *Phys. Rev. D* **63** 065007 (2001), hep-th/0008054.
13. G. Dvali, G. Gabadadze, M. Kolanović, F. Nitti, *Phys. Rev. D* **65** 024031 (2002), hep-th/0106058.
14. R. Dick, *Class. Quant. Grav.* **18** R1 (2001), hep-th/0105320.
15. R. Dick, *Actaphys. Polon. B* **32** 3669 (2001), hep-th/0110162.
16. C. Deffayet, *Phys. Lett. B* **502** 199 (2001), hep-th/0010186.
17. C. Deffayet, G. R. Dvali and G. Gabadadze, *Phys. Rev. D* **65** 044023 (2002), astro-ph/0105068.
18. C. Deffayet and S. J. Landau, J. Raux, M. Zaldarriaga and P. Astier, *Phys. Rev. D* **66** 024019 (2002), astro-ph/0201164.
19. J. S. Alcaniz, *Phys. Rev. D* **65** 123514 (2002), astro-ph/0202492.
20. D. Jain, A. Dev and J. S. Alcaniz, *Phys. Rev. D* **66** 083511 (2002), astro-ph/0206224.
21. A. Lue, R. Scoccimarro, G. Starrkman, *Phys. Rev. D* **69** 044005 (2004), astro-ph/0307034.
22. A. Lue, *Physics Reports* **423** 1 (2006), astro-ph/0510068.

23. J. E. Campbell, *A Course of Differential Geometry* (Clarendon, Oxford 1926).
24. L. Magaard, *Zur einbettung riemannscher Raume in Einstein-Raume und konforme-euclidische Raume*, (PhD Thesis, Kiel 1963).
25. H. Y. Liu and B. Mashhoon, *Ann. Phys.* (Leipzig) **4**, 565 (1995).
26. H. Y. Liu and P. S. Wesson, *Astrophys. J.* **562**, 1 (2001), gr-qc/0107093.
27. L. X. Xu, H. Y. Liu and B. L. Wang, *Chin. Phys. Lett.* **20** 995 (2003), gr-qc/0304049.
28. B. L. Wang, H. Y. Liu and L. X. Xu, *Mod. Phys. Lett. A* **19** 449 (2004), gr-qc/0304093.
29. Lixin Xu and Hongya Liu, *Int. J. Mod. Phys. D* **14** 883 (2005), astro-ph/0412241.
30. B. R. Chang et al, *Mod. Phys. Lett. A* **20** 923 (2005), astro-ph/0405084.
31. Tomáš Liko and Paul S. Wesson, *Int. J. Mod. Phys. A* **20** 2037 (2005).
32. H. Y. Liu et al, *Mod. Phys. Lett. A* **20** 1973 (2005), gr-qc/0504021.
33. L. X. Xu, H. Y. Liu and C. W. Zhang, *Int. J. Mod. Phys. D* **15** 215 (2006), astro-ph/0510673.
34. C. Zhang et. al., *Mod. Phys. Lett. A* **21** 571 (2006), astro-ph/0602414.
35. Y. Ping et. al. *Int. J. Mod. Phys. A* **22**, 985 (2007) gr-qc/0610094.
36. Sanjeev S. Seahra, *Phys. Rev. D* **68**, 104027 (2003), hep-th/0309081.
37. J. Ponce de Leon, *Mod. Phys. Lett. A* **16**, 2291 (2001), gr-qc/0111011.
38. H. Y. Liu, *Phys. Lett. B* **560** 149 (2003), hep-th/0206198.
39. P. Binétruy, C. Deffayet, U. Ellwanger, D. Langlois, *Phys. Lett. B* **477** 285 (2000).